Prova 2 - Turma 1 (Noite) - 21/08/2024

1) b) $f(x,y) = 4xy^2 - 2x^2y - x$

$$\nabla f(x,y) = (4y^2 - 4xy - 1,\ 8xy - 2x^2)$$

$$\nabla f(x,y) = (0,0) \Leftrightarrow \begin{cases} 4y^2 - 4xy - 1 = 0 \\ 8xy - 2x^2 = 0 \end{cases}$$

$$8xy - 2x^2 = 0 \Rightarrow 2x(4y - x) = 0$$
$$\Rightarrow x = 0 \text{ ou } x = 4y$$

- $x = 0$

$$4y^2 - 4 \cdot 0 \cdot y - 1 = 0 \Rightarrow 4y^2 = 1 \Rightarrow y = \pm \frac{1}{2}$$

- $x = 4y$

$$4y^2 - 4 \cdot 4y \cdot y - 1 = 0 \Rightarrow -12y^2 = 1 \Rightarrow y^2 = -\frac{1}{12}$$
$$\nexists\, y \in \mathbb{R}.$$

Pontos críticos: $(0, 1/2)$, $(0, -1/2)$

$$H(x,y) = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$$

$$= \begin{vmatrix} -4y & 8y-4x \\ 8y-4x & 8x \end{vmatrix}$$

- $H(0, 1/2) = \begin{vmatrix} -2 & 4 \\ 4 & 0 \end{vmatrix} = -16$

$(0, 1/2)$ é ponto de sela

- $H(0, -1/2) = \begin{vmatrix} 2 & -4 \\ -4 & 0 \end{vmatrix} = -16$

$(0, -1/2)$ é ponto de sela

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2) b) $f(x,y) = xy + 7x + 1$

$F(x,y,z) = xy + 7x + 1 - z$

$F(x,y,z) = 0$

Plano tangente em $P(a,b,c)$

$\nabla F(a,b,c)\,(x-a,\, y-b,\, z-c) = 0$

$(b+7,\, a,\, -1)(x-a,\, y-b,\, z-c) = 0$

$\nabla F(a,b,c) \parallel v,$

onde v é o vetor normal ao plano xy $(z=0)$.

$v = (0,0,1)$

logo

$(b+7,\, a,\, -1) \parallel (0,0,1)$

Daí

$(b+7,\, a,\, -1) = -1\,(0,0,1) \Rightarrow$

$b+7=0 \Rightarrow b=-7$

$a=0$

$\therefore$ a equação do plano tangente procurado será:

$(b+7, a, -1)(x-a, y-b, z-c)=0$

$(0,0,-1)(x-0, y+7, z-f(0,-7))=0$

$-z+f(0,-7)=0$

$-z+1=0 \Rightarrow \boxed{z=1}$ //

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3) $D=\{(x,y)\in\mathbb{R}^2 : x^2+y^2\leq 1\}$

$T(x,y)=x^2+2y^2-x$

- No interior do disco

$\nabla T(x,y)=(2x-1, 4y)$

$\nabla T(x,y)=(0,0) \Rightarrow \begin{cases} 2x=1 \Rightarrow x=1/2 \\ 4y=0 \Rightarrow y=0 \end{cases}$

Ponto crítico: $(1/2, 0) \in D$

$$T(1/2,0) = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$$

- Na fronteira: $g(x,y) = x^2 + y^2$

$$\begin{cases} \nabla T(x,y) = \lambda \nabla g(x,y) \\ g(x,y) = 1 \end{cases}$$

$$\begin{cases} (2x-1, 4y) = \lambda (2x, 2y) \\ x^2 + y^2 = 1 \end{cases}$$

$$\begin{cases} 2x - 1 = 2\lambda x \\ 4y = 2\lambda y \\ x^2 + y^2 = 1 \end{cases}$$

$$4y - 2\lambda y = 0 \Rightarrow 2y(2-\lambda) = 0$$

$$\Rightarrow y = 0 \text{ ou } \lambda = 2$$

- $y=0$

$x^2 = 1 \Rightarrow x = \pm 1$

- $\lambda = 2$

$2x - 1 = 4x \Rightarrow 2x = -1 \Rightarrow x = -1/2$

$x^2 + y^2 = 1 \Rightarrow \frac{1}{4} + y^2 = 1 \Rightarrow$

$y^2 = \frac{3}{4} \Rightarrow y = \pm \frac{\sqrt{3}}{2}$

Pontos:

$(1, 0), (-1, 0), (-1/2, \sqrt{3}/2), (-1/2, -\sqrt{3}/2)$

$T(1,0) = 0$

$T(-1,0) = 2$

$T(-1/2, \sqrt{3}/2) = \frac{1}{4} + \frac{3}{2} + \frac{1}{2} = \frac{9}{4}$

$T(-1/2, -\sqrt{3}/2) = \frac{9}{4}$

Portanto,

pontos de maior aquecimento

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right), \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

ponto de menor aquecimento

$\left(\frac{1}{2}, 0\right)$

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4) $f(x,y) = e^{x^2-y^2}$

a) $\nabla f(x,y) = \left(2x e^{x^2-y^2}, -2y e^{x^2-y^2}\right)$

Direção da variação máxima:

$\nabla f(1,1) = (2,-2)$

Taxa de variação máxima

$$\|\nabla f(1,2)\| = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

b) $D_v f(1,2) = \nabla f(1,2) . \dfrac{v}{\|v\|}$

$$= (2,-2)\ (1,3) . \frac{1}{\sqrt{10}}$$

$$= \frac{1}{\sqrt{10}} (2-6)$$

$$= \frac{-4}{\sqrt{10}}$$

c) $f_{xx}(x,y) = 2e^{x^2-y^2} + 4x^2 e^{x^2-y^2}$

$$f_{xx}(1,1) = 2+4 = 6$$

$$f_{xy}(x,y) = -4xy e^{x^2-y^2}$$

$f_{xy}(1,1) = -4$

$f_{yy}(x,y) = -2e^{x^2-y^2} + 4y^2 e^{x^2-y^2}$

$f_{yy}(1,1) = -2 + 4 = 2$